MATH 4X03: Home Assignment # 1

Due to: September 26, 2000

Problem 1: Given a function of two real variables $u = u(x, y)$, construct, if possible, a holomorphic function $f = f(z)$ of complex variable $z = x + iy$. If impossible, explain why.

$$\begin{array}{ll} \text{(a)} & u = \dfrac{y}{x^2 + y^2} \\ \text{(b)} & u = 2x(c - y) \\ \text{(c)} & u = x^3 - y^3 \end{array}$$

Problem 2: Consider the integral

$$I_\epsilon = \int_C z^\alpha f(z) dz,$$

where $\alpha > -1$, α is real, $f(z)$ is a holomorphic function inside C, and C is a Jordan contour:

$$C = \{z : |z| = \epsilon\}$$

Prove that $\lim_{\epsilon \to 0} I_\epsilon = 0$.

Problem 3: Evaluate the integral

$$\int_C \frac{e^{iz} dz}{(z - \pi/2)(z - 3\pi/2)},$$

for two Jordan contours

$$\begin{array}{ll} \text{(a)} & C = \{z : |z| = \pi\} \\ \text{(b)} & C = \{z : |z| = 2\pi\} \end{array}$$

Problem 4: Use power series to evaluate the integrals

$$\begin{array}{ll} \text{(a)} & \displaystyle\int_C \frac{\sin z dz}{z^2}, \\ \text{(b)} & \displaystyle\int_C \frac{(\cos z - 1) dz}{z^3}, \end{array}$$

where C is a unit circle.