MATH 3J04: Solutions to Home Assignment # 6

Problem 22.7 #18: Let X be number of customers per minute. Then, the average of X is $\mu = 120/60 = 2$ (customers per minute). Use the Poisson distribution,

$$f(x) = \frac{2^x}{x!}e^{-2}, \quad x = 0, 1, 2, ...$$

The probability that more than 4 customers have to wait is $P(X > 4) = 1 - P(X \leq 4)$, where

$$P(X \leq 4) = e^{-2}\left[1 + \frac{2}{1} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!}\right] = 0.947,$$

i.e. $P(X > 4) = 0.053$, or 5.3%.

Problem 22.8 #12: Let X be breaking strength in kg. Then, X is said to have the normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where $\mu = 1500$ and $\sigma = 50$. The maximum load is defined by the condition $P(X < X_{\text{max}}) = 0.05$, which lead to the equation

$$\Phi\left(\frac{X_{\text{max}} - \mu}{\sigma}\right) = 0.05.$$

Using Table A8 of App. 5, one can find $X_{\text{max}} = \mu - 1.645\sigma = 1417.8$(kg).

Problem 23.3 #4: From the given sample, one can find $\bar{x} = 10.25$, $n = 8$. The standard deviation is assumed to be known: $\sigma = 1.2$. The standardized variable

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

is normally distributed. Its 95% confidence interval is between $-1.96 \leq Z \leq 1.96$, that gives

$$\bar{x} - 1.96\sigma/\sqrt{n} \leq \mu \leq \bar{x} + 1.96\sigma/\sqrt{n},$$

or $9.42 \leq \mu \leq 11.08$.

Problem 23.3 #10: From the given sample, one can find $\bar{x} = 659.2$, $s = 4.26$, and $n = 8$. The standardized variable

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

has a t-distribution of degree 4. Its 99% confidence interval is between $-4.6 \leq Z \leq 4.6$, that gives

$$\bar{x} - 4.6s/\sqrt{n} \leq \mu \leq \bar{x} + 4.6s/\sqrt{n},$$

or $650.44 \leq \mu \leq 667.96$.

NOTE: The following two problems will NOT be marked. The solutions are described below. These two problems are beyond the complexity of the given Math3J4 course.

Problem 23.4 #12: Let X be the number of cases cured in $n = 400$ cases. The standard medication cures 75% of patients, i.e. $\mu = 0.75n = 300$. Use the binomial distribution for X with $p = 0.75$ and $n = 400$ to estimate $\sigma^2 = np(1-p) = 75$, i.e. $\sigma = 8.66$. The standardized variable

$$Z = \frac{\bar{x} - \mu}{\sigma}$$

is normally distributed with $\bar{x} = 310$ (given). The new medication is considered not to be better if $P(Z \le c) = 95\%$ with 5% of error. The value of c is found from Table A8 of App. 5 as $c = 1.645$. Computing the current value of Z, i.e.

$$Z_{\text{sample}} = \frac{310 - 300}{8.66} = 1.15 < c$$

we conclude that the new medication is not better. (Notice that $310/400 = 77.5\% > 75\%$ but still we can not accept statistically that the new medication is better.)

Problem 23.4 #14: A sample is given with $s = 3.5$ (hours) and $n = 28$. The standardized variable

$$Y = (n-1)\frac{s^2}{\sigma^2}$$

has a chi-square distribution of degree $n - 1$ (see p.1115 of the textbook). If $\sigma < \sigma_0 = 5$ (hours), it is less expensive to replace all batteries simultaneously. Otherwise, i.e. for $\sigma > \sigma_0$, it is less expensive to replace each battery individually. From the condition $P(Y \ge c) = 95\%$ and Table A10 of App.5, one can find $c = 16.2$. Thus, with 5% of error, the value of σ is

$$\sigma < \sqrt{n-1}s/\sqrt{c} = 4.52 < \sigma_0$$

Thus, we conclude that it is less expensive to replace all batteries simultaneously.