MATH 3J04: Solutions to Home Assignment # 5

Problem 19.4 #4: The method is based on the finite difference for the Laplace equation

$$u_{n+1,k} + u_{n-1,k} + u_{n,k+1} + u_{n,k-1} - 4u_{n,k} = 0; \quad n = 1, 2; \quad k = 1, 2.$$

The system for four interior points has the exact solution: $u_{1,1} = -2$, $u_{1,2} = -11$, $u_{2,1} = 2$, and $u_{2,2} = -16$.

Problem 19.6 #6: The explicit method for the heat equation on the given grid is

$$u_{n,k+1} = \frac{1}{2}u_{n,k} + \frac{1}{4}\left(u_{n+1,k} + u_{n-1,k}\right); \quad n = 1, 2, 3, 4; \quad k = 0, 1, 2, 3, 4.$$

The numerical values found from this method are:

k/n	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$k = 0$	0.2	0.4	0.4	0.2
$k = 1$	0.2	0.35	0.35	0.2
$k = 2$	0.19	0.31	0.31	0.19
$k = 3$	0.17	0.28	0.28	0.17
$k = 4$	0.16	0.25	0.25	0.16
$k = 5$	0.14	0.23	0.23	0.14

Problem 19.6 #8: The same explicit method as in the previous problem is complimented by different boundary conditions:

$$u_{5,k} = \sin(50\pi t_k/3), \quad u_{0.k+1} = \frac{1}{2}\left(u_{0,k} + u_{1,k}\right)$$

The numerical values found from this method are:

k/n	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$k = 0$	0	0	0	0	0	0
$k = 1$	0	0	0	0	0	0.5
$k = 2$	0	0	0	0	0.125	0.87
$k = 3$	0	0	0	0.03	0.28	1
$k = 4$	0	0	0.008	0.08	0.40	0.87
$k = 5$	0	0.002	0.025	0.14	0.44	0.5

Problem 19.7 #2: The explicit method for the wave equation on the given grid is

$$u_{n,k+1} = u_{n-1,k} + u_{n+1,k} - u_{n,k-1}; \quad n = 1, 2, 3, 4; \quad k = 0, 1, 2, 3, 4.$$

The first step should be computed as

$$u_{n,1} = \frac{1}{2}\left(u_{n-1,0} + u_{n+1,0}\right), \quad n = 1, 2, 3, 4.$$

The numerical values found from this method are:

k/n	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$k = 0$	0.032	0.096	0.144	0.128
$k = 1$	0.048	0.088	0.112	0.072
$k = 2$	0.056	0.064	0.016	-0.016
$k = 3$	0.16	-0.016	-0.064	-0.056
$k = 4$	-0.072	-0.112	-0.088	-0.048
$k = 5$	-0.128	-0.144	-0.096	-0.032

Problem 22.3 #6: By computing the number of possible outcomes to get one, two and three Six out of three dices, one can find

$$P = \frac{1}{6^3} + \frac{3 * 5}{6^3} + \frac{3 * 5^2}{6^3} = \frac{91}{216}$$

Problem 22.5 #14: The probability distribution for number of time until the first Six appears:

$$f(x) = \frac{5^{n-1}}{6^n}, \quad x = n$$

The normalization condition is satisfied by using the geometric series

$$\sum_{n=1}^{\infty} p_n = \frac{1}{5}\left(\sum_{n=0}^{\infty}\left(\frac{5}{6}\right)^n - 1\right) = \frac{1}{5}\left(\frac{1}{1 - 5/6} - 1\right) = 1.$$