MATH 3J04: Solutions to Home Assignment # 4

Problem 11.3 #4: The problem has a solution in the form of the Fourier sine-series:

$$u(x,t) = \sum_{n=1}^{\infty} B_n \cos(nt)\sin(nx)$$

where

$$B_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx = \frac{0.4}{\pi n^3}\left(1-(-1)^n\right).$$

Problem 11.4 #19: The boundary conditions are satisfied by the Fourier series:

$$u(x,t) = \sum_{n=0}^{\infty} b_n(t)\sin\left(\frac{\pi(1+2n)x}{2L}\right)$$

The time-evolution problem and the initial conditions are satisfied by

$$b_n(t) = A_n \cos\left(\frac{\pi(1+2n)t}{2L}\right),$$

where A_n is the Fourier sine coefficient of the given function $f(x)$:

$$A_n = \frac{2}{L}\int_0^{L} f(x)\sin\left(\frac{\pi(1+2n)x}{2L}\right)dx$$

Problem 11.5 #4: The solution is a single term of the Fourier sine series for the heat equation:

$$u(x,t) = ke^{-(0.2\pi)^2 t}\sin(0.2\pi x)$$

Problem 11.5 #18(a): The solution is a single term of the Fourier sine series for the Laplace equation:

$$u(x,t) = \frac{\sinh(\pi y)}{\sinh(2\pi)}\sin(\pi x)$$

Problem 11.6 #2: The Fourier Cosine transform for the heat equation is

$$u(x,t) = \int_0^{\infty} A(\omega)e^{-\omega^2 t}\cos(\omega x)d\omega,$$

where $A(\omega)$ is the Fourier transform of the initial condition:

$$A(\omega) = \frac{2}{\pi}\int_0^{\infty} f(x)\cos(\omega x)dx = e^{-\omega}, \quad \omega > 0$$

Problem 11.8 #12: The solution is a single term of the double Fourier sine series for the two-dimensional wave equation:

$$u(x,y,t) = k\cos(\sqrt{2}\pi t)\sin(\pi x)\sin(\pi y)$$