MATH 3J04: Home Assignment # 3

Due to: October 24, 2000

Note: Numbers for problems refer to the main textbook, e.g. problem 7.1: #14 stands for exercise # 14 from section 7.1. Screen or graphical outputs of computer programs such as Matlab programs are allowed provided they are accompanied by clear explanation and details of the method.

Problem 19.1 #6: Solve the initial problem for the logistic equation by the improved Euler (predictor-corrector) method for 10 steps.

$$\frac{dy}{dt} = y - y^2, \quad y(0) = 0.5, \quad \Delta t = 0.1$$

Verify the solution $y(t) = 1/(1 + e^{-t})$. Compute the error.

Problem 19.3 #4: Solve the initial value problem by the Euler method for 5 steps:

$$\frac{d^2y}{dt^2} = t\frac{dy}{dt} - 3y, \quad y(0) = 0, \quad y'(0) = -3, \quad \Delta t = 0.05.$$

Verify the solution $y(t) = t^3 - 3t$. Compute the error.

Problem 19.3 #8: Solve the initial value problem for the Bessel equation by the Runge-Kutta method for 10 steps:

$$t\frac{d^2y}{dt^2} + \frac{dy}{dt} + ty = 0, \quad y(1) = 0.765198, \quad y'(1) = -0.440051, \quad \Delta t = 0.5.$$

Plot the graph of the function, which is the fundamental Bessel function $y(t) = J_0(t)$.

Problem 10.3 #6: Find the Fourier series of the function $f(x) = 1 - x^2$ at $x \in [-1, 1]$ that is extended periodically with the period of 2.

Problem 10.8 #16: Represent the function $f(x) = \pi - x$ for $x \in [0, \pi]$ and $f(x) = 0$ for $x \in [\pi, \infty)$ in the form of the Fourier Sine Transform.

Problem 10.10 #8: Find the Fourier transform of the function $f(x) = e^x$ for $x < 0$ and $f(x) = e^{-x}$ for $x > 0$.