MATH 3J04: Home Assignment # 1

Due to: September 26, 2000

Note: Numbers for problems refer to the main textbook, e.g. problem 7.1: #14 stands for exercise # 14 from section 7.1. Screen or graphical outputs of computer programs such as Matlab programs are allowed provided they are accompanied by clear explanation and details of the method.

Problem 7.1 #14: Find the spectrum and eigenvectors of the matrix:

$$\begin{pmatrix} 0 & 7 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Problem 7.3 #12: Matrix A is called *non-singular* if $\det A \neq 0$. Do there exist non-singular skew-symmetric $3x3$ matrices? $4x4$? $5x5$? Prove it or give details!

Problem 7.5 #10: Find a basis of eigenvectors and diagonalize the matrix:

$$\begin{pmatrix} -19 & 7 \\ -42 & 16 \end{pmatrix}$$

Problem 18.1 #12: Solve the system by the Gauss elimination algorithm:

$$\begin{aligned} 5x_1 + 3x_2 + x_3 &= 2 \\ -4x_2 + 8x_3 &= -3 \\ 10x_1 - 6x_2 + 26x_3 &= -0 \end{aligned}$$

Problem 18.2 #4: Find the LU- factorization of the system and solve it by Doolittle's method:

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= -2 \\ 4x_1 + 5x_2 + 13x_3 &= -7 \\ 10x_1 + 14x_2 + 43x_3 &= -25 \end{aligned}$$

Problem 18.3 #6: Find an approximate solution of the system by doing 3;5;10 iterations of the Gauss–Seidel method. Is the method convergent for this system?

$$\begin{aligned} 4x_1 + 5x_3 &= 12.5 \\ x_1 + 6x_2 + 2x_3 &= 18.5 \\ 8x_1 + 2x_2 + x_3 &= -11.5 \end{aligned}$$