Dot and vector products can be used to solve geometric problems, associated to lines, planes, triangles, parallelograms, and parallelepiped.

Example: Find the shortest distance between two parallel lines:

$$x = 2 + t, \qquad x = 1 + s,$$
$$y = -1 - t, \qquad y = -s,$$
$$z = 3 + 4t \qquad z = 1 + 4s$$

Example: Find the shortest distance from a point $P(x, y, z)$ to the line defined by the point $P_0(x_0, y_0, z_0)$ and the direction vector $\mathbf{d}$.

A **parallelepiped** is spanned by three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$, which do not belong to the same plane in the three-dimensional space. Volume of a parallelepiped is a product of the area of a base and the length of its height.

<u>Theorem:</u> The volume of the parallelepiped spanned by vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is $|\mathbf{w} \cdot (\mathbf{u} \times \mathbf{w})|$.

<u>Example:</u> Find the volume of the parallelepiped spanned by

$$\mathbf{u} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} -4 \\ -3 \\ -2 \end{pmatrix}$$