Vectors

A **vector** is an arrow between two points, referred to as the **initial** and **terminal** points. The distance between the initial and terminal points for a vector $\mathbf{a}$ is called the **length** (or **magnitude** or **norm**) of the vector and it is denoted by $\|\mathbf{a}\|$.

Properties of a vector

1. Two vectors are **equal** if they have the same length and the same direction.
2. The vector is **zero** if it has the zero length.
3. Two vectors are **parallel** if they have the same or opposite directions.
4. The vector is **negative** of another vector if it is obtained by exchanging the initial and terminal points. The negative vector has the same length and the opposite direction.

Vectors are added using the **parallelogram rule**.

Properties of vector addition

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$
3. $\mathbf{v} + \mathbf{0} = \mathbf{v}$
4. $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$

Example: Show that

$$\mathbf{v} + (\mathbf{u} - \mathbf{v}) = \mathbf{u}$$

The **scalar multiple** $k\mathbf{v}$ of a vector $\mathbf{v}$ by a scalar k is the vector that is parallel to $\mathbf{v}$ in the same direction if $k > 0$ or in the opposite direction if $k < 0$ and it has the length $|k|\|\mathbf{v}\|$.

Properties of scalar multiplication

1. $\|k\mathbf{v}\| = |k|\|\mathbf{v}\|$
2. $1\mathbf{v} = \mathbf{v}$
3. $0\mathbf{v} = \mathbf{0}$, $k\mathbf{0} = \mathbf{0}$

Example: Prove that the diagonals of a parallelogram intersect at exactly mid-points.

A vector $\mathbf{v}$ is called a **unit** vector if it has the unit length: $\|\mathbf{v}\| = 1$.

<u>Example</u>

If $\mathbf{v} \neq \mathbf{0}$, then $\frac{1}{\|\mathbf{v}\|}\mathbf{v}$ is a unit vector in the same direction as $\mathbf{v}$.

<u>Theorem</u>: Two vectors are parallel if and only if one vector is a scalar multiple of the other vector.