Any matrix can be brought to reduced row-echelon form by a sequence of elementary row operations. The number of leading 1's in the reduced row-echelon form is called the **rank** of the matrix.

Theorem: Consider a linear system of m equations for n variables. Let r be the rank of the augmented matrix and r_0 be the rank of the coefficient matrix.

- If $r_0 < r$, the system is inconsistent.
- If $r_0 = r$, the system is consistent and the set of solutions has exactly $(n - r)$ parameters.

Example:

Characterize solutions of the linear system for different values of numbers (a, b, c):

$$\begin{aligned} x + 2y + z &= a \\ 2x + 4y + 2z &= b \\ 3x + 6y + 3z &= c \end{aligned}$$

When all constant terms are zero, the linear system is called **homogeneous**. If at least one constant term is non-zero, the linear system is called **inhomogeneous**.

A homogeneous system always has a zero solution, which is called the **trivial** solution. If a solution has at least one non-zero component, it is called a **non-trivial** solution.

Theorem: Consider a linear system of m equations for n variables. Let r be the rank of the coefficient matrix.

- If $r = n$, the homogeneous system has only the trivial solution.
- If $r < n$, there exist infinitely many non-trivial solutions with exactly $(n - r)$ parameters.

Example:

Solve the following homogeneous system:

$$\begin{aligned} 2x_1 + 3x_2 - x_4 &= 0 \\ x_1 + x_3 + x_4 &= 0 \end{aligned}$$